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Instantons and Holomorphic Couplings in Intersecting D-brane Models

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Abstract

We clarify certain aspects and discuss extensions of the recently introduced string D-instanton calculus (hep-th/0609191). The one-loop determinants are related to one-loop open string threshold corrections in intersecting D6-brane models. Utilising a non-renormalisation theorem for the holomorphic Wilsonian gauge kinetic functions, we derive a number of constraints for the moduli dependence of the matter field Kähler potentials of intersecting D6-brane models on the torus. Moreover, we compute string one-loop corrections to the Fayet-Iliopoulos terms on the D6-branes finding that they are proportional to the gauge threshold corrections. Employing these results, we discuss the issue of holomorphy for E2-instanton corrections to the superpotential. Eventually, we discuss E2-instanton corrections to the gauge kinetic functions and the FI-terms.

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1 Introduction

Type IIA orientifolds with intersecting D6-branes and their mirror symmetric Type IIB counterparts have proven to provide a phenomenologically interesting class of string compactifications and have been under intense investigation during the last couple of years [1, 2, 3, 4].

In order to make contact with experiment one needs not only the means to determine the gauge group and chiral matter content of such a string model, but also has to develop tools for the computation of the low energy effective action. It is in this latter low energy description where important issues like moduli stabilisation and supersymmetry breaking are discussed.

In this paper we would like to clarify certain important aspects of this effective action, which to our knowledge have so far not been spelled out in the literature. The first issue concerns the properties of the gauge couplings in $\mathcal{N} = 1$ supersymmetric D6-brane vacua. The physical one-loop open string threshold corrections to the gauge couplings have been computed in [5, 6] for toroidal backgrounds. Here we first state a non-renormalisation theorem for the holomorphic gauge kinetic function at the one-loop level and then explicitly show that it is indeed satisfied for the Wilsonian gauge couplings in toroidal Type IIA orientifolds.

We revisit perturbative one-loop corrections to the Fayet-Iliopoulos (FI) terms for intersecting D6-branes. In [7] it was shown that, if the D6-branes are su-

persymmetric at tree-level, in a globally consistent model no such corrections are generated at one-loop. We ask the question whether a small non-vanishing FI-term on a D6_b-brane can induce an FI-term on a D6_a-brane which is supersymmetric at tree-level. We find the intriguing result that the one-loop induced FI-term on brane D6_a can be expressed by the gauge threshold corrections.

Moving back to gauge couplings, the extraction of the Wilsonian part involves an interesting interplay between the non-holomorphic gauge couplings and the Kähler potentials for all the matter fields involved, providing strong constraints on the complete moduli dependence of the matter field Kähler potentials. As we will see, in order to cancel all σ -model anomalies in the effective action, a one-loop redefinition of the dilaton S -field as well as of the complex structure moduli U is needed. The hereby induced corrections to the gauge coupling constants will be referred to as “universal” threshold corrections (in analogy with the heterotic string). This is in contrast to perturbative heterotic string compactifications, where only the dilaton acquires a universal one-loop field redefinition [8, 9, 10, 11, 12, 13].

Having revisited and discussed the perturbative one-loop corrections, in the second part of the paper we undertake some first steps towards a better understanding of possible D-instanton effects. Such effects are very important for an understanding of the vacuum structure (these days called the landscape) of string compactifications and it has been pointed out recently that they can also generate phenomenologically appealing terms like Majorana masses for neutrinos [14, 15, 16, 17, 18, 19, 20]. Moreover, they are also important for the string theory description of gauge instanton effects [21, 22, 23, 24, 25] (see [26] for a recent general review on instantons).

String instantons are given by wrapped string world-sheets as well as by wrapped Euclidean D-branes and, like in field theory, their contributions to the space-time superpotential are quite restricted. These contributions can be computed in a semi-classical approach, i.e. one involving only the tree-level instanton action and a one-loop determinant for the fluctuations around the instanton [27]. For type IIA orientifold models on Calabi–Yau spaces with intersecting D6-branes (and their T-dual cousins) the contribution of wrapped Euclidean D2-branes, hereafter called E2-branes, to the superpotential has been determined in [14] (see also [15]). Since both the D6-branes and the E2-instantons are described by open string theories, it was shown that (in the spirit of the D(−1) instantons treated e.g. in [28, 29, 21]) the entire instanton computation boils down to the evaluation of disc and one-loop string diagrams with boundary (changing) operators inserted. Here both the D6-branes and the E2-instantons wrap compact three-cycles of the Calabi–Yau manifold.

Intriguingly, the one-loop contributions in the instanton amplitude [14] have been shown to be identical to string threshold corrections for the gauge couplings of the corresponding D6-branes [17, 23]. This relates the computation of such instanton amplitudes to the discussion in the first half of this paper. So far it has

not been explained explicitly in which sense the computed instantonic correlation functions are meant to be holomorphic. With the results from the first part of this paper, we clarify this point.

Finally, we show that E2-instantons not only contribute to the superpotential but, from the zero mode counting, can also contribute to the holomorphic gauge kinetic functions for the $SU(N)$ gauge groups localised on the D6-branes. In order for such corrections to arise, the E2-instanton must not be rigid but must admit one extra pair of fermionic zero modes arising from a deformation of the instanton. This is the space-time instanton generalisation of a fact known from topological string theory, namely that world-sheet instantons induce $\text{tr}(W^2)^{h-1}$ couplings if they have h boundaries. We will see that such couplings can also arise from space-time E2-instantons. Finally, we find that the zero mode counting also allows E2-instanton corrections to the FI-terms on the D6-branes. Similar to the one-loop corrections, these can arise once the supersymmetry on the E2-brane is softly broken by for instance turning on the C_3 -form modulus through the world-volume of E2.

2 A non-renormalisation theorem

Let us investigate the structure of perturbative and non-perturbative corrections to the holomorphic gauge kinetic functions for Type II orientifolds. We discuss this for Type IIA orientifolds, but this is of course related via mirror symmetry to the corresponding Type IIB orientifolds.

Consider a Type IIA orientifold with O6-planes and intersecting D6-branes preserving $\mathcal{N} = 1$ supersymmetry in four dimensions, i.e. the D6-branes wrap special Lagrangian (sLag) three-cycles Π_a of the underlying Calabi–Yau manifold \mathcal{X} , all preserving the same supersymmetry. On the threefold we introduce in the usual way a symplectic basis (A_I, B^I) , $I = 0, 1, \dots, h_{2,1}$ of homological three-cycles with the topological intersection numbers

$$A_I \circ B^J = \delta_I^J. \quad (2.1)$$

Moreover, we assume that the A_I cycles are invariant under the orientifold projection and that the B^J cycles are projected out. The complexified complex structure moduli on such an orientifold are defined as

$$U_I^c = \frac{1}{(2\pi) \ell_s^3} \left[e^{-\phi_4} \int_{A_I} \Re(\widehat{\Omega}_3) - i \int_{A_I} C_3 \right], \quad (2.2)$$

where $\widehat{\Omega}_3$ denotes the normalised holomorphic three-form on \mathcal{X} and the four-dimensional dilaton is defined by $\phi_4 = \phi_{10} - \frac{1}{2} \ln(V_{\mathcal{X}}/\ell_s^6)$. Expanding a three cycle Π_a into the symplectic basis,

$$\Pi_a = M_a^I A_I + N_{a,I} B^I, \quad (2.3)$$

with $M^I, N_I \in \mathbb{Z}$, from dimensional reduction of the Dirac-Born-Infeld (DBI) action one can deduce the $SU(\mathcal{N}_a)$ gauge kinetic functions at string tree-level

$$f_a = \sum_{I=0}^{h_{2,1}} M_a^I U_I^c. \quad (2.4)$$

Since the imaginary parts of the U_I^c are axionic fields, they enjoy a Peccei-Quinn shift symmetry $U_I^c \rightarrow U_I^c + c_I$ which is preserved perturbatively and only broken by E2-brane instantons.

Let C_i denote a basis of anti-invariant 2-cycles, i.e. $C_i \in H_-^{1,1}$. The complexified Kähler moduli are then defined as

$$T_i^c = \frac{1}{\ell_s^2} \left(\int_{C_i} J_2 - i \int_{C_i} B_2 \right), \quad (2.5)$$

where B_2 denotes the NS-NS two-form of the Type IIA superstring. Therefore, also the complexified Kähler moduli enjoy a Peccei-Quinn shift symmetry, broken by world-sheet instantons. Note, that the chiral fields T_i^c organise the σ -model perturbation theory and do not contain the dilaton, so that the string perturbative theory is entirely defined by powers of the U_I^c . Moreover, to shorten the notation we denote by U_I^c and T_i^c the complexified moduli and by U_I and T_i only the real parts.

The superpotential W and the gauge kinetic function f in the four-dimensional effective supergravity action are holomorphic quantities. In the usual way, employing holomorphy and the Peccei-Quinn symmetries above, one arrives at the following two non-renormalisation theorems.

The superpotential can only have the following dependence on U_I^c and T_i^c

$$W = W_{\text{tree}} + W^{\text{NP}}(e^{-U_I^c}, e^{-T_i^c}), \quad (2.6)$$

i.e. beyond tree-level there can only be non-perturbative contributions from world-sheet and E2-brane instantons. Similarly, the holomorphic gauge kinetic function must look like

$$f_a = \sum_I M_a^I U_I^c + f_a^{\text{1-loop}}(e^{-T_i^c}) + f_a^{\text{NP}}(e^{-U_I^c}, e^{-T_i^c}), \quad (2.7)$$

i.e. in particular its one-loop correction must not depend on the complex structure moduli. Finally, we consider the Fayet-Iliopoulos terms for the $U(1)_a$ gauge fields on the D6-branes. At string tree-level and for small deviations from the supersymmetry locus, these are given by

$$\xi_a = e^{-\phi_4} \int_{\Pi_a} \Im(\widehat{\Omega}_3) = e^{-\phi_4} N_a^I \int_{B_I} \Im(\widehat{\Omega}_3) \quad (2.8)$$

and therefore only depend on the complex structure moduli. At this classical level there are no α' corrections. It is an important question about brane stability whether these FI-terms receive perturbative or non-perturbative corrections in g_s . Again, non-renormalisation theorems say that in the Wilsonian sense one expects perturbative corrections at most at one-loop.

In the following, we will be concerned with the terms beyond tree-level appearing in (2.6), (2.7) and for the FI-terms. First, we discuss the one-loop threshold corrections $f^{1\text{-loop}}(e^{-T_i^c})$, which also make their appearance in the space-time instanton generated superpotential $W^{\text{np}}(e^{-U_I^c}, e^{-T_i^c})$. Second, we will revisit the computation of stringy one-loop corrections to the FI-terms. Finally, we will discuss $f^{\text{np}}(e^{-U_I^c}, e^{-T_i^c})$ as well as instanton corrections to the FI-terms.

3 One-loop thresholds for intersecting D6-branes on T^6

The purpose of this section is to recall the one-loop results for the gauge threshold corrections in intersecting D6-brane models [5, 30, 6]. The gauge coupling constants of the various gauge group factors G_a in such a model, up to one loop, have the form

$$\frac{8\pi^2}{g_a^2(\mu)} = \frac{8\pi^2}{g_{a,\text{string}}^2} + \frac{b_a}{2} \ln\left(\frac{M_s^2}{\mu^2}\right) + \frac{\Delta_a}{2}, \quad (3.1)$$

where b_a is the beta function coefficient. The first term corresponds to the gauge coupling constant at the string scale, which contains the tree-level gauge coupling as well as the “universal” contributions at one-loop (see section 5.2). These contributions are universal in the sense that they originate from a redefinition of the dilaton and complex structure moduli at one-loop. The redefinition is brane stack and therefore gauge group independent. However, as the gauge couplings differ for the various gauge groups already at tree level, this correction effectively is gauge group dependent. The second term gives the usual one-loop running of the coupling constants, and the third term denotes the one-loop string threshold corrections originating from integrating out massive string excitations. The last two terms can be computed as a sum of all annulus and Möbius diagrams with one boundary on brane a in the presence of a background magnetic field in the four-dimensional space-time:

$$\begin{aligned} b_a \ln\left(\frac{M_s^2}{\mu^2}\right) + \Delta_a &= \sum_b T^A(\text{D6}_a, \text{D6}_b) + \sum_{b'} T^A(\text{D6}_a, \text{D6}_{b'}) \\ &\quad + T^A(\text{D6}_a, \text{D6}_{a'}) + T^M(\text{D6}_a, \text{O6}). \end{aligned} \quad (3.2)$$

Here, $\text{D6}_{c'}$ denotes the orientifold image of brane c . In an orbifold one also has to take into account the orbifold images of the branes and orientifold planes.

The relevant amplitudes for the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold have been computed [5, 6]. In a sector preserving $\mathcal{N} = 1$ supersymmetry (this means in particular $\sum_I \theta_{ab}^I = 0$) the annulus and Möbius amplitudes are (after subtracting terms which upon summing over all diagrams vanish due to the tadpole cancellation condition) [6]

$$T^A(D6_a, D6_b) = \frac{I_{ab} N_b}{2} \left[\ln \left(\frac{M_s^2}{\mu^2} \right) \sum_{I=1}^3 \text{sign}(\theta_{ab}^I) - \ln \prod_{I=1}^3 \left(\frac{\Gamma(|\theta_{ab}^I|)}{\Gamma(1 - |\theta_{ab}^I|)} \right)^{\text{sign}(\theta_{ab}^I)} - \sum_{I=1}^3 \text{sign}(\theta_{ab}^I) (\ln 2 - \gamma) \right], \quad (3.3)$$

$$T^M(D6_a, O6_k) = \pm I_{a;O6_k} \left[\ln \left(\frac{M_s^2}{\mu^2} \right) \sum_{I=1}^3 \text{sign}(\theta_{a;O6_k}^I) - \ln \prod_{I=1}^3 \left(\frac{\Gamma(2|\theta_{a;O6_k}^I|)}{\Gamma(1 - 2|\theta_{a;O6_k}^I|)} \right)^{\text{sign}(\theta_{a;O6_k}^I)} + \sum_{I=1}^3 \text{sign}(\theta_{a;O6_k}^I) (\gamma - 3 \ln 2) \right], \quad (3.4)$$

where I_{ab} is the intersection number of branes a and b , N_b is the number of branes on stack b and $\pi \theta_{ab}^I$ is the intersection angle of branes a and b on the I 'th torus. Similarly, $I_{a;O6_k}$ denotes the intersection number of brane a and orientifold plane k and $\pi \theta_{a;O6_k}^I$ their intersection angle. The formula for T^M is only valid for $|\theta_{a;O6_k}^I| < 1/2$, the formulas for other cases look similar [6].

In a sector preserving $\mathcal{N} = 2$ supersymmetry one finds [5]

$$T^A(D6_a, D6_b) = N_b |I_{ab}^J I_{ab}^K| \left[\ln \left(\frac{M_s^2}{\mu^2} \right) - \ln |\eta(i T_I^c)|^4 - \ln(T_I V_I^a) + \gamma_E - \ln(4\pi) \right], \quad (3.5)$$

where I denotes the torus on which the branes lie on top of each other, T_I its Kähler modulus and T_I^c its complexification with $T_I = \Re(T_I^c)$. Furthermore, $V_I^a = |n_a^I + i u_I m_a^I|^2 / u_I$, with u_I the complex structure modulus of the torus and n_a^I, m_a^I the wrapping numbers on the I 'th torus. Note that the moduli dependence of the one-loop threshold function in the $\mathcal{N} = 2$ sectors is in complete agreement with the non-renormalisation theorems of section two (see eq. (2.7)), since the holomorphic part of $\Delta_a^{\mathcal{N}=2}$ is proportional only to $\ln \eta(i T_I^c)$.

For the $\mathcal{N} = 1$ sectors, the one-loop thresholds in a given open string D6-brane sector have the following form (specialising to the case $\theta_{ab}^{1,2} > 0, \theta_{ab}^3 < 0$):

$$\Delta_a = -\frac{b_a}{16\pi^2} \ln \left[\frac{\Gamma(\theta_{ab}^1)\Gamma(\theta_{ab}^2)\Gamma(1 + \theta_{ab}^3)}{\Gamma(1 - \theta_{ab}^1)\Gamma(1 - \theta_{ab}^2)\Gamma(-\theta_{ab}^3)} \right]. \quad (3.6)$$

This expression is a non-holomorphic function of the complex structure moduli U_I^c . Hence, for the $\mathcal{N} = 1$ sectors, the holomorphic one-loop gauge kinetic function $f_a^{1\text{-loop}}(e^{-T_i^c})$ vanishes. The emergence of the non-holomorphic terms in the one-loop threshold corrections will be further discussed in section 5.

4 Fayet-Iliopoulos terms

In this section we investigate one-loop corrections to the FI-terms for a $U(1)_a$ gauge field on the D6_a-brane induced by the presence of other branes D6_b. Such corrections for Type I string vacua have already been studied in [31, 32] and the case of intersecting D6-branes has been discussed in [7]. Here we are following essentially the computational technique of [7]. The crucial observation is that the vertex operator for the auxiliary D-field in the (0)-ghost picture is simply given by the internal world-sheet $U(1)$ current, i.e. $V_{D_a}^{(0)} = J_{U(1)}$. Therefore, the one-point function of $V_{D_a}^{(0)}$ on the annulus with boundaries a and b can be written as

$$\langle V_{D_a} \rangle = -\frac{i}{2\pi} \partial_\nu \int_0^\infty dt Z_{ab}(\nu, it)|_{\nu=0}, \quad (4.1)$$

where $Z_{ab}(\nu, it)$ denotes the annulus partition function, with insertion of $\exp(2\pi i J_0)$, in the open string sector (ab) , where J_0 is the zero mode of the $U(1)$ current. In the case of intersecting D6-branes on a torus preserving $\mathcal{N} = 1$ supersymmetry and after application of the Riemann theta-identities, this partition function is given by

$$Z_{ab}(\nu, it) = I_{ab} N_b \frac{(-i)^3}{\pi^4 t^2} \frac{\vartheta_1(\frac{3\nu}{2}, it) \prod_I \vartheta_1(-\frac{\nu}{2} + i\frac{\theta_I}{2}t, it)}{\eta^3(it) \prod_I \vartheta_1(i\frac{\theta_I}{2}t, it)}. \quad (4.2)$$

Using that $\vartheta_1(0, it) = 0$ and $\vartheta'_1(\nu, it)|_{\nu=0} = -2\pi\eta^3$, one obtains the divergence

$$\langle V_{D_a} \rangle \simeq I_{ab} N_b \int_0^\infty \frac{dt}{t^2}, \quad (4.3)$$

which is cancelled by tadpole cancellation in a global model. Therefore, once the D6-branes are supersymmetric at tree-level, no FI-term is generated at one-loop level and the system is not destabilised. This result is consistent with the computations in [31, 32].

However, this is not the end of the story of the one-loop corrections to FI-terms in intersecting D6-brane models. One can also envision that a tree-level FI-term on a brane D6_b induces via a one-loop diagram an FI-term on a brane D6_a. To proceed we assume that $\theta_b^I \rightarrow \theta_b^I + 2\epsilon^I$ with $\sum \epsilon^I = \epsilon$ and compute

$$\langle V_{D_a} \rangle_{\epsilon^I} = -\frac{i}{2\pi} \partial_\nu \partial_{\epsilon^I} \int_0^\infty dt Z_{ab}(\epsilon^I, \nu, it)|_{\nu=0, \epsilon^I=0} \quad (4.4)$$

with

$$Z_{ab}(\epsilon^I, \nu, it) = I_{ab} N_b \frac{(-i)^3}{\pi^4 t^2} \frac{\vartheta_1(\frac{3\nu}{2} + i\frac{\epsilon^I}{2}t, it) \prod_I \vartheta_1(-\frac{\nu}{2} + i\frac{(\epsilon^I - \epsilon^J - \epsilon^K)}{2}t + i\frac{\theta_I}{2}t, it)}{\eta^3(it) \prod_I \vartheta_1(i\epsilon^I t + i\frac{\theta_I}{2}t, it)}.$$

The derivative with respect to the supersymmetry breaking parameters ϵ^I brings down one factor of t and it turns out that the result is the same for all ∂_{ϵ^I} :

$$\langle V_{D_a} \rangle_\epsilon \simeq i I_{ab} N_b \int_0^\infty \frac{dt}{t} \sum_{I=1}^3 \frac{\vartheta'_1}{\vartheta_1} \left(\frac{i\theta_{ab}^I t}{2}, \frac{it}{2} \right). \quad (4.5)$$

This is the same expression as the one-loop threshold corrections $T^A(D6_a, D6_b)$, so that at linear order in ϵ we obtain for the FI-terms

$$(\xi_a)_\epsilon = (\epsilon_b - \epsilon_a) T^A(D6_a, D6_b). \quad (4.6)$$

Completely analogously, one can show that this formula remains true also for $\mathcal{N} = 2$ open string sectors. Therefore, we would like to propose that such a relation between gauge threshold and one-loop corrections to FI-terms is valid for general intersecting D6-brane models. Moreover, the Wilsonian part of the thresholds $T^A(D6_a, D6_b)$, which we compute in the next section, should also be the Wilsonian part of the correction to the FI-terms.

5 Wilsonian gauge kinetic function and σ -model anomalies

In a supersymmetric gauge theory one can compute the running gauge couplings $g_a(\mu^2)$ in terms of the gauge kinetic functions f_a , the Kähler potential \mathcal{K} and the Kähler metrics of the charged matter fields $K^{ab}(\mu^2)$ [33, 8, 9, 10, 11, 12, 13]:

$$\begin{aligned} \frac{8\pi^2}{g_a^2(\mu^2)} &= 8\pi^2 \Re(f_a) + \frac{b_a}{2} \ln \left(\frac{\Lambda^2}{\mu^2} \right) + \frac{c_a}{2} \mathcal{K} \\ &\quad + T(G_a) \ln g_a^{-2}(\mu^2) - \sum_r T_a(r) \ln \det K_r(\mu^2), \end{aligned} \quad (5.1)$$

with

$$b_a = \sum_r n_r T_a(r) - 3 T(G_a), \quad c_a = \sum_r n_r T_a(r) - T(G_a) \quad (5.2)$$

and $T_a(r) = \text{Tr}(T_{(a)}^2)$ ($T_{(a)}$ being the generators of the gauge group G_a). In addition, $T(G_a) = T_a(\text{adj } G_a)$ and n_r is the number of multiplets in the representation r of the gauge group and the sums run over these representations. In this context, the natural cutoff scale for a field theory is the Planck scale, i.e. $\Lambda^2 = M_{\text{Pl}}^2$.

The left hand side of eq. (5.1) is given by eq. (3.1), which contains the gauge coupling at the string scale, $1/g_{a,\text{string}}^2$, as well as the one-loop string threshold corrections Δ_a . In general, Δ_a is the sum of a non-holomorphic term plus the real-part of a holomorphic threshold correction:

$$\Delta_a = \Delta_a^{\text{n.h.}} + \Re(\Delta_a^{\text{hol.}}). \quad (5.3)$$

On the right hand side of eq. (5.1), f_a denotes the Wilsonian, i.e. holomorphic, gauge kinetic function, which is given in terms of a holomorphic tree-level function plus the holomorphic part of the one-loop threshold corrections (cf. eq. (2.7)):

$$f_a = f_a^{\text{tree}} + f_a^{\text{1-loop}}(e^{-T_i^c}) = \sum_I M_a^I U_I^c + \Delta_a^{\text{hol.}}. \quad (5.4)$$

In addition, on the right hand side of eq. (5.1) the non-holomorphic terms proportional to the Kähler metric of the moduli \mathcal{K} and the matter field Kähler metrics K_r are due to the one-loop contributions of massless fields. These fields generate non-local terms in the one-loop effective action, which correspond to one-loop non-invariances under σ -model transformations, the so-called σ -model anomalies (Kähler and reparametrisation anomalies).

Matching up all terms in eq. (5.1) essentially means that the σ -model anomalies can be cancelled in a two-fold way. First, by local contributions to the gauge coupling constant via the one-loop threshold contributions Δ_a . These terms originate from massive string states. The second way to cancel the σ -model anomalies is due to a field dependent (however gauge group independent) one-loop contribution to the Kähler potential of the chiral moduli fields. It implies that some of the moduli fields transform non-trivially under the Kähler transformations and also under reparametrisations in moduli space. The universal one-loop modification of the Kähler potential is nothing else than a generalised Green-Schwarz mechanism cancelling the σ -model anomalies. This is analogous to the Green-Schwarz mechanism which cancels anomalies of physical $U(1)$ gauge fields, whereas the σ -model anomalies correspond to unphysical, composite gauge connections. Effectively it means that the Green-Schwarz mechanism with respect to the σ -model anomalies can be described by a non-holomorphic, one-loop field redefinition of the associated tree-level moduli fields.

As we will see, in type IIA orientifold models these field redefinitions act on the real parts of the dilaton field S as well as the complex structure moduli U_J :

$$\begin{aligned} S &\rightarrow S + \delta^{GS}(U, T) \\ U_J &\rightarrow U_J + \delta_J^{GS}(U, T). \end{aligned} \quad (5.5)$$

These redefined fields are those that determine the gauge coupling constants $1/g_{a,\text{string}}^2$ at the string scale. Recall that, as the tree-level gauge couplings (2.4) are already gauge group dependent, so are these one-loop corrections, but the only dependence arises due to the universal one-loop redefinition of the moduli fields. It is in this sense that we still call these one-loop corrections to the gauge couplings universal. Note that in heterotic string compactifications, the σ -model Green-Schwarz mechanism only acts on the heterotic dilaton field.

5.1 Holomorphic gauge couplings for toroidal models

In summary, equation (5.1) is to be understood recursively, which means that one can insert the tree-level results into the last three terms of eq. (5.1). In addition, one also has to include the universal field redefinition eq. (5.5) in $1/g_{a,\text{string}}^2$ in the left hand side of eq. (5.1), in order to get a complete matching of all terms in eq. (5.1), as we will demonstrate for the aforementioned toroidal orbifold in the following. For $\mathcal{N} = 2$ sectors the one-loop threshold corrections to the gauge coupling constant indeed contain a holomorphic, Wilsonian term $f_a^{(1)}$, whereas for $\mathcal{N} = 1$ sectors only the non-holomorphic piece $\Delta_a^{\text{n.h.}}$ is present.

Specifically, the holomorphic gauge kinetic function can now be determined by comparing the string theoretical formula (3.1) for the effective gauge coupling with the field theoretical one (5.1). The first thing to notice are the different cutoff scales appearing in the two formulas. One needs to convert one into the other using

$$\frac{M_s^2}{M_{\text{Pl}}^2} \propto \exp(2\phi_4) \propto (S U_1 U_2 U_3)^{-\frac{1}{2}}. \quad (5.6)$$

Here, ϕ_4 is the four dimensional dilaton and the complex structure moduli in the supergravity basis can be expressed in terms of ϕ_4 and the complex structure moduli $u_I = R_{I,2}/R_{I,1}$ as

$$S = \frac{1}{2\pi} e^{-\phi_4} \frac{1}{\sqrt{u_1 u_2 u_3}}, \quad U_I = \frac{1}{2\pi} e^{-\phi_4} \sqrt{\frac{u_J u_K}{u_I}}, \text{ with } I \neq J \neq K \neq I. \quad (5.7)$$

These fields are the real parts of complex scalars of four dimensional chiral multiplets S^c and U_I^c .

As $\mathcal{N} = 4$ super Yang–Mills theory is finite, one expects the sum of the terms in (3.1) proportional to $T(G_a)$ to cancel. This is because the only chiral multiplets transforming in the adjoint representation of the gauge group are the open string moduli which (on the background considered) assemble themselves into three chiral multiplets, thus forming an $\mathcal{N} = 4$ sector together with the gauge fields. To show that this cancellation does happen, one notices the following. Firstly, $n_{\text{adjoint}} = 3$, as explained, such that there is no term in b_a proportional to $T(G_a)$. Secondly,

$$\mathcal{K} = -\ln(S^c + \bar{S}^c) - \sum_{I=1}^3 \ln(U_I^c + \bar{U}_I^c) - \sum_{I=1}^3 \ln(T_I^c + \bar{T}_I^c) \quad (5.8)$$

$$g_{a,\text{tree}}^{-2} = S \prod_{I=1}^3 n_a^I - \sum_{I=1}^3 U_I n_a^I m_a^J m_a^K \quad I \neq J \neq K \neq I, \quad (5.9)$$

where T_I are the Kähler moduli of the torus and n_a^I, m_a^I are the wrapping numbers of the brane. Finally, one needs the matter metric for the open string moduli,

which can be obtained from the T-dual expression in models with D9- and D5-branes [4]. Performing the T-duality, which essentially amounts to exchanging Kähler and complex structure moduli and converting gauge flux into non-trivial intersection angles for the D6-branes, one arrives at ($I = 1, \dots, 3$)¹:

$$K_{ij}^I = \frac{\delta_{ij}}{T_I U_I} \left| \frac{(n_a^J + i u_J m_a^J)(n_a^K + i u_K m_a^K)}{(n_a^I + i u_I m_a^I)} \right| \quad I \neq J \neq K \neq I. \quad (5.10)$$

Let us now turn to the fields in the fundamental representation of the gauge group G_a , in particular to the fields arising from the intersection with one other stack of branes, denoted by b . For an $\mathcal{N} = 1$ open string sector the metric for these fields can be written as [34, 4] (see also [35])

$$K_{ij}^{ab} = \delta_{ij} S^{-\alpha} \prod_{I=1}^3 U_I^{-(\beta + \xi \theta_{ab}^I)} T_I^{-(\gamma + \zeta \theta_{ab}^I)} \sqrt{\frac{\Gamma(\theta_{ab}^1)\Gamma(\theta_{ab}^2)\Gamma(1 + \theta_{ab}^3)}{\Gamma(1 - \theta_{ab}^1)\Gamma(1 - \theta_{ab}^2)\Gamma(-\theta_{ab}^3)}}, \quad (5.11)$$

where $\alpha, \beta, \gamma, \xi$ and ζ are undetermined constants. As $\theta_{ab}^{1,2} > 0$ and $\theta_{ab}^3 < 0$, which is assumed in (5.11), the intersection number I_{ab} is positive, implying that

$$n_f = I_{ab} N_b. \quad (5.12)$$

Using $T_a(f) = \frac{1}{2}$ and relations (5.2, 5.6, 5.8, 5.11, 5.12) one finds a contribution to the right hand side of (5.1) proportional to

$$\begin{aligned} & \frac{I_{ab} N_b}{2} \left(\ln \left(\frac{M_s^2}{\mu^2} \right) + (2\gamma - 1) \ln(T_1 T_2 T_3) + (2\beta - \frac{1}{2}) \ln(U_1 U_2 U_3) + (2\alpha - \frac{1}{2}) \ln S \right. \\ & \left. + \zeta \sum_{I=1}^3 \theta_{ab}^I \ln T_I + \xi \sum_{I=1}^3 \theta_{ab}^I \ln U_I - \ln \left[\frac{\Gamma(\theta_{ab}^1)\Gamma(\theta_{ab}^2)\Gamma(1 + \theta_{ab}^3)}{\Gamma(1 - \theta_{ab}^1)\Gamma(1 - \theta_{ab}^2)\Gamma(-\theta_{ab}^3)} \right] \right). \end{aligned} \quad (5.13)$$

Using (3.6) one finds that the first and the last term exactly reproduce the contribution of the last two terms in (3.1). The terms proportional to ζ and ξ will later be shown to constitute the aforementioned universal gauge coupling correction. The remaining three terms can neither be attributed to such a correction nor can they be written as the real part of a holomorphic function. Thus they cannot be the one-loop correction to the gauge kinetic function and therefore must vanish. This fixes some of the coefficients in the ansatz (5.11):

$$\alpha = \beta = \frac{1}{4}, \quad \gamma = \frac{1}{2}. \quad (5.14)$$

¹An overall factor involving the wrapping numbers was introduced in this expression in order to achieve full cancellation. This can be done, as the expressions used are derived only up to overall constants [34].

The same matching of terms appears between the Möbius diagram plus the annulus with boundaries on brane a and its orientifold image and the Kähler metrics for fields in the symmetric and antisymmetric representation. Here, one has to replace θ_{ab}^I and $I_{ab}N_b$ by $\theta_{aa'}^I = 2\theta_a^I$ and $I_{aa'}N_a$ in (5.11) and (3.3). Apart from these replacements, the Kähler metric for matter in these representations is also given by (5.11) with the constants α, β, γ given in (5.14).

The corrections to the gauge couplings coming from $\mathcal{N} = 2$ open string sectors were seen in the previous section to take on quite a different form. They contain a term,

$$-\ln |\eta(iT_I^c)|^4 = -4 \operatorname{Re} [\ln \eta(iT_I^c)], \quad (5.15)$$

which can be written as the real part of a holomorphic function. This leads one to conclude that the gauge kinetic function receives one-loop corrections from these sectors. Inserting the correct prefactor, which from the first term in (3.5) and the corresponding one in (5.1) can be seen to be proportional to the beta function coefficient, gives

$$f_a^{(1)} = -\frac{N_b |I_{ab}^J I_{ab}^K|}{4\pi^2} \ln \eta(iT_I^c) \quad I \neq J \neq K \neq I, \quad (5.16)$$

where again I denotes the torus in which the branes lie on top of each other and $I_{ab}^{J,K}$ are the intersection numbers on the other tori.

The term $-\ln(T_I V_I^a)$ in (3.5) is not the real part of a holomorphic function. Proceeding as before, one finds that the Kähler metric for the hypermultiplet (or two chiral multiplets) living at an intersection of branes a and b preserving eight supercharges must be

$$K_{ij}^I = \frac{|n_a^I + i u_I m_a^I|}{(U^J U^K T^J T^K)^{\frac{1}{2}}} \quad I \neq J \neq K \neq I. \quad (5.17)$$

Apart from the factor in the numerator, this is in agreement with the form found by direct calculations [34, 4]. The appearance of the numerator is however plausible as it also appears in the open string moduli metric and the hypermultiplets under discussion should feel the I 'th torus in the same way.

5.2 Universal threshold corrections

In the following, the aforementioned ‘‘universal’’ gauge coupling corrections will be discussed. They also appear in the heterotic [13] and type I [36, 37] string and are related to a redefinition of the dilaton at one-loop [9, 8]. This stems from the fact that the dilaton really lives in a linear multiplet rather than a chiral one.

Our general ansatz for the Kähler metrics for the chiral matter in an $\mathcal{N} = 1$ sector contains a factor

$$\prod_{J=1}^3 U_J^{-\xi \theta_{ab}^J} T_J^{-\zeta \theta_{ab}^J}, \quad (5.18)$$

which according to (5.1) appears in the one-loop correction to the gauge coupling constant. Neither is this term reproduced in the string one-loop calculation of the coupling nor can it be written as a correction to the holomorphic gauge kinetic function. Therefore, as is familiar from gauge threshold computations, there remains the possibility that it can be absorbed into a one-loop correction to the S and U_I chiral superfields. In the following, we require that such a gauge group factor independent universal correction is possible and see how this fixes the parameters in (5.18).

The first observation is, that in order to get something gauge group independent, the factor (5.18) actually must have the following form

$$\prod_{J=1}^3 U_J^{-\xi' \operatorname{sign}(I_{ab})\theta_{ab}^J} T_J^{-\zeta' \operatorname{sign}(I_{ab})\theta_{ab}^J}, \quad (5.19)$$

with ξ' and ζ' independent of the brane. For the metrics of fields transforming in the symmetric or antisymmetric representation of the gauge group, one has to replace $\phi_{ab} = \phi_a - \phi_b$ by $\phi_{aa'} = 2\phi_a$ and $\operatorname{sign}(I_{ab})$ by $\operatorname{sign}(I_{aa'} - I_{a;O6})$ or $\operatorname{sign}(I_{aa'} + I_{a;O6})$, respectively. Then one computes (K' denotes the factor (5.18, 5.19) appearing in the full Kähler metric K):

$$\begin{aligned} \sum_r T_a(r) \ln \det K'^r &= \frac{|I_{ab}|N_b}{2} \ln \left[\prod_{J=1}^3 U_J^{-\xi' \operatorname{sign}(I_{ab})\theta_{ab}^J} T_J^{-\zeta' \operatorname{sign}(I_{ab})\theta_{ab}^J} \right] \\ &\quad + \frac{|I_{ab'}|N_b}{2} \ln \left[\prod_{J=1}^3 U_J^{-\xi' \operatorname{sign}(I_{ab'})\theta_{ab'}^J} T_J^{-\zeta' \operatorname{sign}(I_{ab'})\theta_{ab'}^J} \right] \\ &\quad + \frac{N_a+2}{2} \frac{|I_{aa'}-I_{a;O6}|}{2} \ln \left[\prod_{J=1}^3 U_J^{-2\xi' \operatorname{sign}(I_{aa'}-I_{a;O6})\theta_a^J} T_J^{-2\zeta' \operatorname{sign}(I_{aa'}-I_{a;O6})\theta_a^J} \right] \\ &\quad + \frac{N_a-2}{2} \frac{|I_{aa'}+I_{a;O6}|}{2} \ln \left[\prod_{J=1}^3 U_J^{-2\xi' \operatorname{sign}(I_{aa'}+I_{a;O6})\theta_a^J} T_J^{-2\zeta' \operatorname{sign}(I_{aa'}+I_{a;O6})\theta_a^J} \right]. \end{aligned} \quad (5.20)$$

After a few steps, using $|I_{ab}| \operatorname{sign}(I_{ab}) = I_{ab}$ and the tadpole cancellation condition, this can be brought to the simple form

$$\begin{aligned} \sum_r T_a(r) \ln \det K'^r &= -n_a^1 n_a^2 n_a^3 \left[\sum_b N_b m_b^1 m_b^2 m_b^3 \sum_{I=1}^3 \theta_b^I (\xi' \ln U_I + \zeta' \ln T_I) \right] \\ &\quad - \sum_{J=1}^3 n_a^J m_a^K m_a^L \left[\sum_b N_b m_b^J n_b^K n_b^L \sum_{I=1}^3 \theta_b^I (\xi' \ln U_I + \zeta' \ln T_I) \right] \\ &\quad J \neq K \neq L \neq J. \end{aligned} \quad (5.21)$$

Therefore, these corrections have precisely the form required for them to be identified with the one-loop correction between the linear superfields appearing in

string theory and the chiral superfields used in the supergravity description

$$\begin{aligned} S^L &= S - \frac{1}{8\pi^2} \sum_b N_b m_b^1 m_b^2 m_b^3 \sum_{I=1}^3 \phi_b^I (\xi' \ln U_I + \zeta' \ln T_I) \\ U_J^L &= U_J + \frac{1}{8\pi^2} \sum_b N_b m_b^J n_b^K n_b^L \sum_{I=1}^3 \phi_b^I (\xi' \ln U_I + \zeta' \ln T_I) \\ &\quad J \neq K \neq L \neq J. \end{aligned} \quad (5.22)$$

In contrast to eq. (5.9), where the tree-level gauge couplings are determined, the one-loop gauge couplings at the string scale have to include the redefined fields S^L and U^L :

$$g_{a,\text{string}}^{-2} = S^L \prod_{I=1}^3 n_a^I - \sum_{I=1}^3 U_I^L n_a^I m_a^J m_a^K. \quad (5.23)$$

In contrast to all (to us) known cases studied in the literature, for $\xi' \neq 0$ the fields which are corrected, i.e. the moduli S and U_I , also appear in the one-loop redefinition. Let us propose an argument, why such corrections might be expected to be absent: Due to the anomalous $U(1)$ gauge symmetries, the chiral superfields S and U_I participate in the Green-Schwarz mechanism and therefore transform non-trivially under $U(1)$ gauge transformations. This implies that, in order to be gauge invariant, the one-loop corrections in (5.22) proportional to $\ln U_I$ must be extended in the usual way by $U_I \rightarrow U_I + \delta_{GS}^a V_a$. Computing the resulting FI-terms via the supergravity formula $\xi_a/2g_a^2 = \partial K/\partial V_a|_{V_a=0}$ gives, besides the tree-level result depending on S^L, U_I^L , a one-loop contribution proportional to $\xi' \sum_b \xi_b^{(0)} \phi_b^J / U_J$. This has an extra dependence on the complex structure moduli U_I . However, for intersecting D6-branes, we have seen that the Wilsonian (supergravity) FI-Terms are proportional to the Wilsonian gauge threshold corrections, which depend only on the Kähler moduli via instanton corrections (for $\mathcal{N} = 1$ sectors they are even vanishing in the setup at hand). This seems to suggest that there should better be no $U_I^{-\theta_I}$ dependence in the matter field Kähler metrics, i.e. $\xi' = 0$.

Moreover, in analogy to the heterotic string we expect that for the range $-1 \leq \theta_J \leq 1$ the exponent of T_J runs over the range $[-1, 0]$. This condition would fix $\zeta' = \pm 1/2$.

Let us summarise the conclusions we have drawn from requiring holomorphy of the Wilsonian gauge kinetic function. First, we provided arguments that the Kähler metric for $\mathcal{N} = 1$ chiral matter fields in intersecting D6-brane models is of the following form

$$K_{ij}^{ab} = \delta_{ij} S^{-\frac{1}{4}} \prod_{J=1}^3 U_J^{-\frac{1}{4}} T_J^{-\left(\frac{1}{2} \pm \frac{1}{2} \text{sign}(I_{ab}) \theta_{ab}^J\right)} \sqrt{\frac{\Gamma(\theta_{ab}^1) \Gamma(\theta_{ab}^2) \Gamma(1 + \theta_{ab}^3)}{\Gamma(1 - \theta_{ab}^1) \Gamma(1 - \theta_{ab}^2) \Gamma(-\theta_{ab}^3)}}, \quad (5.24)$$

where supersymmetry of course requires $\sum_{I=1}^3 \theta_{ab}^I = 0$. Second, the holomorphic gauge kinetic function (on the background considered) only receives corrections from $\mathcal{N} = 2$ open string sectors and the one-loop correction takes on the following form

$$f_a^{(1)} = - \sum_b \frac{N_b |I_{ab}^J I_{ab}^K|}{4\pi^2} \ln \eta(i T_I^c) \quad I \neq J \neq K \neq I, \quad (5.25)$$

where the sum only runs over branes b which lie on top of brane a in exactly one torus, denoted by I . Therefore, the results for the gauge threshold corrections and the matter field Kähler metrics are consistent both with the non-renormalisation theorem from section 2 and the Kaplunovsky-Louis formula (5.1). Clearly, it would be interesting, along the lines of [34] to carry out a string amplitude computation to fix the free coefficient in the ansatz (5.11) and see whether our indirect arguments are correct.

6 Holomorphic E2-instanton amplitudes

Space-time instantons are given also by D-branes, which in this case are Euclidean D2-branes (so-called E2-branes) wrapping three-cycles Ξ in the Calabi–Yau, so that they are point-like in four-dimensional Minkowski space. Such instantons can contribute to the holomorphic superpotential and gauge kinetic functions only if they preserve half of the $\mathcal{N} = 1$ supersymmetry. This means that the instanton measure must contain a factor $d^4x d^2\theta$. Let us first clarify an important aspect of this half-BPS condition. In the second part of this section we then revisit the computation of contributions of such instantons to the superpotential and also clarify some issues concerned with the appearing one-loop determinants. In the third and fourth part, we investigate under which conditions such string instantons can also contribute to the gauge kinetic functions and FI-terms.

6.1 Half-BPS instantons

As has been explained in [24, 18, 20], just wrapping an E2-instanton around a rigid sLag three-cycle in the Calabi-Yau gives four bosonic and four fermionic zero modes. The vertex operators for the latter are

$$V_\theta^{(-1/2)}(z) = \theta_\alpha e^{-\frac{\varphi(z)}{2}}(z) S^\alpha(z) \Sigma_{h=\frac{3}{8}, q=\frac{3}{2}}(z) \quad (6.1)$$

and

$$V_{\overline{\theta}}^{(-1/2)}(z) = \overline{\theta}_{\dot{\alpha}} e^{-\frac{\varphi(z)}{2}}(z) S^{\dot{\alpha}}(z) \Sigma_{h=\frac{3}{8}, q=-\frac{3}{2}}(z). \quad (6.2)$$

Therefore, if the instanton is not invariant under the orientifold projection, one still has four instead of the desired two fermionic zero modes. Thus, only by

placing the E2-brane in a position invariant under $\Omega\overline{\sigma}$ does one have a chance to get rid of the two additional zero modes $\overline{\theta}$. For so called $O(n)$ instantons one can see that the zero modes x_μ, θ are symmetrised and the mode $\overline{\theta}$ gets anti-symmetrised. For the opposite projection, i.e. for $USp(2n)$ instantons, the zero modes x_μ, θ are anti-symmetrised and the mode $\overline{\theta}$ gets symmetrised. Therefore, one can only get the simple $d^4x d^2\theta$ instanton measure for a single $O(1)$ instanton.

6.2 Superpotential contributions

In order to contribute to the superpotential, we also require that there do not arise any further zero modes from E2-E2 open strings, so that the three-cycle Ξ should be rigid, i.e. $b_1(\Xi) = 0$. Therefore, considering an E2-instanton in an intersecting brane configuration, additional zero modes can only arise from the intersection of the instanton Ξ with D6-branes Π_a . There are $N_a [\Xi \cap \Pi_a]^+$ chiral fermionic zero modes $\lambda_{a,I}$ and $N_a [\Xi \cap \Pi_a]^-$ anti-chiral ones, $\overline{\lambda}_{a,J}$.²

For its presentation it is useful to introduce the short-hand notation

$$\widehat{\Phi}_{a_k,b_k}[\vec{x}_k] = \Phi_{a_k,x_{k,1}} \cdot \Phi_{x_{k,1},x_{k,2}} \cdot \Phi_{x_{k,2},x_{k,3}} \cdot \dots \cdot \Phi_{x_{k,n-1},x_{k,n}} \cdot \Phi_{x_{k,n(k)},b_k} \quad (6.3)$$

for the chain-product of open string vertex operators. Here we define $\widehat{\Phi}_{a_k,b_k}[\vec{0}] = \Phi_{a_k,b_k}$.

To extract the superpotential, one can probe it by evaluating an appropriate matter field correlator in the instanton background. The CFT allows one to compute it in physical normalisation which combines the superpotential part Y with the matter field Kähler metrics like

$$\langle \Phi_{a_1,b_1} \cdot \dots \cdot \Phi_{a_M,b_M} \rangle_{E2\text{-inst}} = \frac{e^{\frac{K}{2}} Y_{\Phi_{a_1,b_1}, \dots, \Phi_{a_M,b_M}}}{\sqrt{K_{a_1,b_1} \cdot \dots \cdot K_{a_M,b_M}}}. \quad (6.4)$$

In [14] a general expression for the single E2-instanton contribution to the charged matter superpotential was proposed involving the evaluation of the following zero mode integral over disc and one-loop open string CFT amplitudes

$$\begin{aligned} \langle \Phi_{a_1,b_1} \cdot \dots \cdot \Phi_{a_M,b_M} \rangle_{E2} &= \frac{V_3}{g_s} \int d^4x d^2\theta \sum_{\text{conf.}} \prod_a (\prod_{i=1}^{[\Xi \cap \Pi_a]^+} d\lambda_a^i) (\prod_{i=1}^{[\Xi \cap \Pi_a]^-} d\overline{\lambda}_a^i) \\ &\exp(-S_{E2}) \exp(Z'_0(E2)) \langle \widehat{\Phi}_{a_1,b_1}[\vec{x}_1] \rangle_{\lambda_{a_1}, \overline{\lambda}_{b_1}} \cdot \dots \cdot \langle \widehat{\Phi}_{a_L,b_L}[\vec{x}_L] \rangle_{\lambda_{a_L}, \overline{\lambda}_{b_L}}. \end{aligned} \quad (6.5)$$

For simplicity, we do not consider the case that matter fields are also assigned to string loop diagrams. The one-loop contributions are annulus diagrams for

²Here we introduced the physical intersection number between two branes $\Pi_a \cap \Pi_b$, which is the sum of positive $[\Pi_a \cap \Pi_b]^+$ and negative $[\Pi_a \cap \Pi_b]^-$ intersections.

open strings with one boundary on the E2-instanton and the other boundary on the various D6-branes and Möbius diagrams with boundary on the E2-instanton

$$\langle 1 \rangle^{\text{1-loop}} = Z'_0(\text{E2}) = \sum_b Z'^A(\text{E2}_a, \text{D6}_b) + Z'^M(\text{E2}_a, \text{O6}) . \quad (6.6)$$

Here Z' means that we only sum over the massive open string states in the loop amplitude, as the zero modes are taken care of explicitly. It was shown that these instantonic open string loop diagrams are identical to the one-loop threshold corrections $T^A(\text{D6}_a, \text{D6}_b)$. Diagrammatically we have the intriguing relation shown in figure 1 and in figure 2, which holds for the even spin structures³.

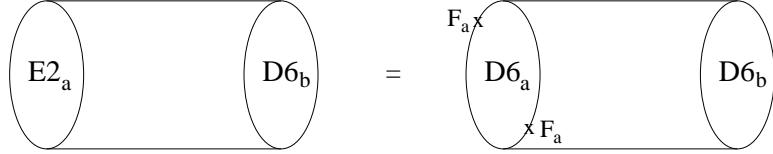


Figure 1: Relation between instantonic one-loop amplitudes and corresponding gauge threshold corrections

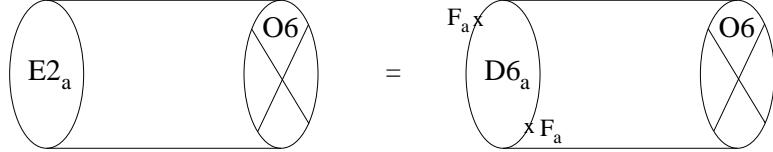


Figure 2: Relation between instantonic Möbius amplitude and corresponding gauge threshold corrections

The annulus threshold corrections can be computed, leading to

$$Z^A(\text{E2}_a, \text{D6}_b) = \int_0^\infty \frac{dt}{t} \sum_{\alpha, \beta \neq (\frac{1}{2}, \frac{1}{2})} (-1)^{2(\alpha+\beta)} \frac{\vartheta''[\alpha]_\beta(it)}{\eta^3(it)} A_{ab}^{\text{CY}}[\alpha]_\beta(it) \quad (6.7)$$

and the Möbius strip amplitude for the instanton, which as we explained must be invariant under the orientifold projection, yields

$$Z^M(\text{E2}_a, \text{O6}) = \pm \int_0^\infty \frac{dt}{t} \sum_{\alpha, \beta \neq (\frac{1}{2}, \frac{1}{2})} (-1)^{2(\alpha+\beta)} \frac{\vartheta''[\alpha]_\beta(it + \frac{1}{2})}{\eta^3(it + \frac{1}{2})} A_{aa}^{\text{CY}}[\alpha]_\beta(it + \frac{1}{2}) \quad (6.8)$$

³The contribution of the CP-odd R^- sector is expected to yield corrections to the θ -angle.

The overall plus sign is for $O(1)$ instantons, reflecting the fact that only for these the x_μ and θ_α zero modes survive the orientifold projection. Note that up to the argument, the Möbius thresholds are $Z^A(E2_a, D6_a)$. Therefore, for rigid branes the massless sector reflects the number of four bosonic and two fermionic zero modes. In section 6.3 we will discuss the number of zero modes if $b_1(\Xi) > 0$. All these stringy threshold corrections are known to be non-holomorphic. Therefore, it is not immediately obvious in which sense the expression (6.5) is meant and how one can extract the holomorphic superpotential part Y from it.

The CFT disc amplitudes in (6.5) are also not holomorphic but combine non-holomorphic Kähler potential contributions and holomorphic superpotential contributions in the usual way [38, 39, 40, 41]:

$$\langle \widehat{\Phi}_{a,b}[\vec{x}] \rangle_{\lambda_a, \bar{\lambda}_b} = \frac{e^{\frac{\mathcal{K}}{2}} Y_{\lambda_a \Phi_{a,x_1} \Phi_{x_1,x_2} \dots \Phi_{x_N,b} \bar{\lambda}_b}}{\sqrt{K_{\lambda_a,a} K_{a,x_1} \dots K_{x_N,b} K_{b,\bar{\lambda}_b}}} \quad (6.9)$$

$$= \frac{e^{\frac{\mathcal{K}}{2}} Y_{\lambda_a \widehat{\Phi}_{a,b}[x] \bar{\lambda}_b}}{\sqrt{K_{\lambda_a,a} \widehat{K}_{a,b}[x] K_{b,\bar{\lambda}_b}}}. \quad (6.10)$$

Due to the Kaplunovsky-Louis formula (5.1), the stringy one-loop amplitudes are known to include the holomorphic Wilsonian part and contributions from wavefunction normalisation. Applied to the instanton one-loop amplitudes appearing in $Z_0(E2_a)$, we write

$$\begin{aligned} Z_0(E2_a) &= -8\pi^2 \Re(f_a^{(1)}) - \frac{b_a}{2} \ln\left(\frac{M_p^2}{\mu^2}\right) - \frac{c_a}{2} \mathcal{K}_{\text{tree}} \\ &\quad - \ln\left(\frac{V_3}{g_s}\right)_{\text{tree}} + \sum_b \frac{|I_{ab}N_b|}{2} \ln [\det K^{ab}]_{\text{tree}}, \end{aligned} \quad (6.11)$$

where for the brane and instanton configuration in question the coefficients are

$$b_a = \sum_b \frac{|I_{ab}N_b|}{2} - 3, \quad c_a = \sum_b \frac{|I_{ab}N_b|}{2} - 1. \quad (6.12)$$

The constant contributions arise from the Möbius amplitude. Inserting (6.9) and (6.11) in (6.5), one realises that the Kähler metrics involving an instanton zero mode and a matter field precisely cancel out, so that only the matter metrics survive, as required by the general form (6.4). Moreover, the term $\exp(\mathcal{K}/2)$ comes out just right due to the rule that each disc contains precisely two instanton zero modes. The holomorphic piece in (6.4) can therefore be expressed entirely in terms of other holomorphic quantities like holomorphic Yukawa couplings, the holomorphic instanton action and the one-loop holomorphic Wilsonian gauge kinetic function on the E2-brane:

$$\begin{aligned} Y_{\Phi_{a_1,b_1}, \dots, \Phi_{a_M,b_M}} &= \sum_{\text{conf.}} \text{sign}_{\text{conf}} \exp(-S_{E2})_{\text{tree}} \exp(-f_a^{(1)}) \\ &\quad Y_{\lambda_{a_1} \widehat{\Phi}_{a_1,b_1}[\vec{x}_1] \bar{\lambda}_{b_1}} \cdot \dots \cdot Y_{\lambda_{a_L} \widehat{\Phi}_{a_L,b_L}[\vec{x}_L] \bar{\lambda}_{b_L}}. \end{aligned} \quad (6.13)$$

This explicitly shows that knowing the tree-level Kähler potentials, computing the matter field correlator in the instanton background up to one-loop level in g_s is sufficient to deduce the Wilsonian holomorphic instanton generated superpotential. Higher order corrections in g_s only come from loop corrections to the Kähler potentials.

6.3 Instanton corrections to the gauge kinetic functions

So far we have discussed space-time instanton corrections to the superpotential. These involved one-loop determinants, which are given by annulus vacuum diagrams with at least one E2 boundary. These are related to one-loop gauge threshold corrections to the gauge theory on a D6-brane wrapping the same cycle as the E2 instanton.

Now we can ask what other corrections these space-time instantons can induce. By applying S- and T-dualities to the story of world-sheet instanton corrections in the heterotic string, we expect that there can also be *E2-instanton corrections* to the holomorphic gauge kinetic functions. In the heterotic case, similar to the topological Type II string, such corrections arise from string world-sheets of Euler characteristic zero, i.e. here from world-sheets with two boundaries. Therefore, we expect such corrections to appear for E2-instantons admitting one complex open string modulus, i.e. those wrapping a three-cycle with Betti number $b_1(\Xi) = 1$.

Let us start by discussing the instanton zero mode structure for such a cycle. First let us provide the form of the vertex operators. The bosonic fields in the (-1) ghost picture are

$$V_y^{(-1)}(z) = y e^{-\varphi(z)} \Sigma_{h=\frac{1}{2}, q=\pm 1}(z) \quad (6.14)$$

which, before the orientifold projection, are accompanied by the two pairs of fermionic zero modes

$$V_\mu^{(-1/2)}(z) = \mu_\alpha e^{-\frac{\varphi(z)}{2}} S^\alpha(z) \Sigma_{h=\frac{3}{8}, q=-\frac{1}{2}}(z) \quad (6.15)$$

and

$$V_{\bar{\mu}}^{(-1/2)}(z) = \bar{\mu}_{\dot{\alpha}} e^{-\frac{\varphi(z)}{2}} S^{\dot{\alpha}}(z) \Sigma_{h=\frac{3}{8}, q=+\frac{1}{2}}(z). \quad (6.16)$$

Now one has to distinguish two cases depending on how the anti-holomorphic involution $\bar{\sigma}$ acts on the open string modulus Y

$$\bar{\sigma} : y \rightarrow \pm y. \quad (6.17)$$

In the case that y is invariant under $\bar{\sigma}$, called first kind in the following, the orientifold projection acts in the same way as for the 4D fields X_μ , i.e. the two bosonic zero modes y and the two fermionic zero modes $\bar{\mu}$ survive. In the

other case, dubbed second kind, the bosonic zero mode is projected out and only the fermionic modulino zero mode μ survives⁴. Therefore, in the absence of any additional zero modes, for instance from E2-D6 intersections, the zero mode measure in any instanton amplitude assumes the following form

$$\int d^4x d^2\theta d^2y d^2\bar{\mu} e^{-S_{E2}} \dots, \quad \text{for } \bar{\sigma} : y \rightarrow y \quad (6.18)$$

and

$$\int d^4x d^2\theta d^2\mu e^{-S_{E2}} \dots, \quad \text{for } \bar{\sigma} : y \rightarrow -y. \quad (6.19)$$

As an example consider the set-up in figure 3 with $\bar{\sigma} : y_i \rightarrow -y_i$. Here the

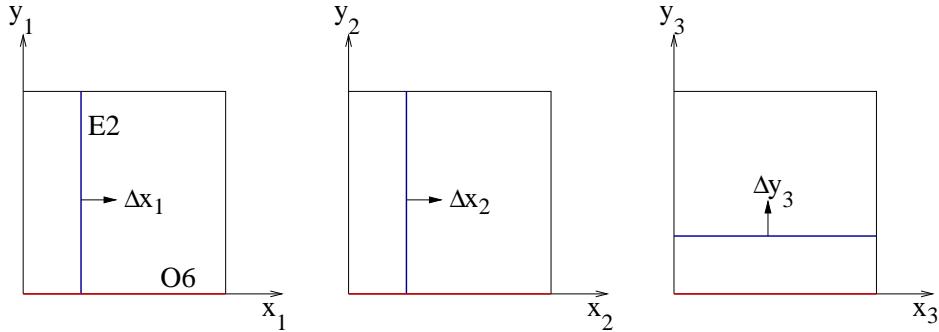


Figure 3: Deformations of an instanton which is invariant under the orientifold projection

deformations $\Delta x_{1,2}$ are of the first kind and Δy_3 is of the second kind.

Now, it is clear that an instanton with precisely one set of fermionic zero modes of the second kind and no additional zero modes can generate a correction to the $SU(N_a)$ gauge kinetic function. The instanton amplitude takes on the following form

$$\langle F_a(p_1) F_a(p_2) \rangle_{E2} = \int d^4x d^2\theta d^2\mu \exp(-S_{E2}) \exp(Z'_0(E2)) A_{F_a^2}(E2, D6_a)$$

where $A_{F_a^2}(E2, D6_a)$ is the annulus diagram in figure 4, which absorbs all the appearing fermionic zero modes and where the gauge boson vertex operators in the (0)-ghost picture have the usual form

$$V_A^{(0)}(z) = \epsilon^\mu (\partial_\mu X(z) + i(p \cdot \psi) \psi_\mu(z)) e^{ip \cdot X(z)}. \quad (6.20)$$

⁴By duality, this distinction is related to the two kinds of deformations of genus g curves studied in [42]. The first kind are the curves moving in families, i.e. transversal deformation of the curve. The second kind is related to the deformations coming with the genus g of the curve.

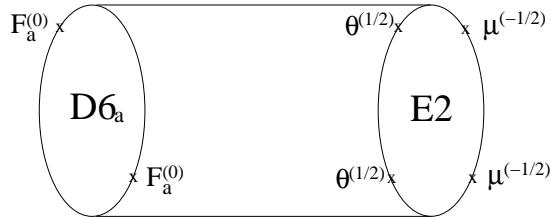


Figure 4: Annulus diagram for E2-instanton correction to f_a . The upper indices give the ghost number of the vertex operators.

Analogous to world-sheet instantons, these diagrams can be generalised to multi $\text{tr}(W^2)^h$ couplings. Just from the zero mode counting one immediately sees that they can be generated by E2-instantons with h sets of complex deformation zero modes of the second kind and no other additional zero modes. Then, besides the annulus diagram in figure 4, there are $h - 1$ similar diagrams. On the $D6_a$ brane one inserts two gauginos in the $(+1/2)$ ghost picture and on the E2 boundary two μ modulinos in the $(-1/2)$ ghost picture. Clearly, once the internal $\mathcal{N} = 2$ superconformal field theory is known, as for toroidal orbifolds or Gepner models, these annulus diagrams can be computed explicitly. They involve up to four-point functions of vertex operators on an annulus world-sheet with the two boundaries on the E2 and the $D6_a$ brane. Very similar to the $\mathcal{N} = 2$ open string sectors for loop-corrections to f_a , one expects these instanton diagrams to also contain a sum over world-sheet instantons. Therefore, the generic E2-instanton contribution to the holomorphic gauge kinetic functions has the moduli dependence $f^{\text{np}}(e^{-U_i^c}, e^{-T_i^c})$.

6.4 Instanton corrections to the FI-terms

Having shown that E2-instanton corrections to the gauge couplings are possible, it is natural to investigate whether such instantons also contribute to the FI-terms for the $U(1)$ gauge symmetries on the $D6$ -branes. As we have seen in section 4, at the one-loop level the contributions to the gauge couplings and to the FI-terms have the same functional form.

Assume now that, as in the last section, in the background with intersecting $D6$ -branes we can find an E2-instanton with only two θ fermionic zero modes and two additional fermionic zero modes related to a deformation of the E2. If now similar to the $D6$ -branes we could break supersymmetry on the E2-branes by a slight deformation of the complex structure, then we would expect four θ -like, four μ -like and two y -like zero modes. As shown in figure 5, these could generate an FI-term on the $D6$ -branes. However, since the E2-brane must be invariant, i.e. an $O(1)$ instanton, under the orientifold projection, a complex structure deformation does not necessarily break supersymmetry on the E2-instanton. In

this case, the analogous situation to the one-loop D6-brane generation of the FI-term cannot happen.

However, there is another mechanism to generate an FI-term on the E2-instanton, namely by turning on the $\int_{\Xi} C_3$ modulus through the three-cycle the E2-instanton is wrapping. This also appears in the (generalised) calibration condition [43, 44] for supersymmetry on the E2-brane. Therefore, it is possible that the one-loop diagram in figure 5 indeed generates an FI-term on the $D6_a$ brane once the C_3 flux through the E2 is non-zero.

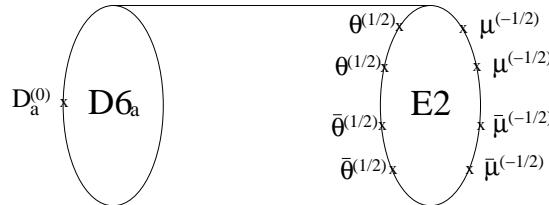


Figure 5: Annulus diagram for E2-instanton correction to ξ_a . The upper indices give the ghost number of the vertex operators.

Here we will leave a further study of the concrete instanton amplitudes for g_a^{-2} and ξ_a and their relation for future work and conclude that just from fermionic zero mode counting, we have evidence that E2-instanton corrections to both the gauge kinetic functions and the FI-terms are likely to appear.

7 Conclusions

In this paper we have investigated a number of aspects related to loop and D-brane instanton corrections to intersecting D6-brane models in Type IIA orientifolds. In particular, we have revisited the computation of one-loop corrections to the FI-terms.

Using results for the gauge threshold corrections in intersecting D6-brane models on a toroidal orientifold, we explicitly computed the Wilsonian holomorphic gauge coupling in this setup. On the way, exploiting holomorphy and the Shifman-Vainshtein, respectively Kaplunovsky-Louis formula, it was possible to constrain the form of the matter field Kähler metrics. In the second part, we discussed E2-brane instanton corrections to the superpotential, the gauge kinetic function and the FI-terms. For the first, we showed in which sense one can extract the form of the holomorphic superpotential from a superconformal field theory correlation function of matter fields in the E2-instanton background.

Moreover, we showed that E2-instantons wrapping a three-cycle which has precisely one complex deformation and no matter zero modes, can in principle

contribute to the gauge kinetic function for a gauge theory on a stack of D6-branes. By turning on the R-R three-form modulus, also instanton corrections to the FI-terms become possible. A more detailed investigation of the appearing annulus diagrams is necessary to eventually establish the appearance of these instanton corrections, but our first steps indicate that such corrections are indeed present in $\mathcal{N} = 1$ D-brane vacua.

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